SUMMER PACKET
for students entering Geometry

DETERMINING WHETHER A POINT IS ON A LINE

Example 1

Decide whether (3, 2) is a solution of the equation \( y = 2x - 8 \)

\[-2 = 2(3) - 8 \quad \text{Substitute 3 for } x \text{ and } -2 \text{ for } y.\]

\[-2 = -2 \quad \text{Simplify.}\]

The statement is true, so (3, 2) is a solution of the equation \( y = 2x - 8 \)

Exercises: Decide whether the given ordered pair is a solution of the equation.

1. \( y = 6x + 4; (-2, 8) \) \( \text{NO} \)
2. \( y = -10x - 2; (1, -12) \) \( \text{YES} \)
3. \( y = -\frac{1}{4}x - 18; (-4, -17) \) \( \text{YES} \)
4. \( y = \frac{3}{2}x + 10; (4, 12) \) \( \text{NO} \)
5. \( y = \frac{5}{9}x + 34; (-9, 27) \) \( \text{NO} \)
6. \( y = \frac{2}{3}x - 6; (9, 0) \) \( \text{YES} \)

CALCULATING SLOPE

Example 2

Find the slope of a line passing through \((3, -9)\) and \((2, -1)\).

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Formula for slope}\]

\[ m = \frac{-1 - (-9)}{2 - 3} = \frac{-8}{1} \quad \text{Substitute values and simplify.}\]

\[ m = \frac{8}{-1} = -8 \quad \text{Slope is } -8.\]

Exercises: Find the slope of the line that contains the points

7. \((4, 1), (3, 6)\) \( \frac{-5}{1} = -5 \)
8. \((-8, 0), (5, -2)\) \( \frac{-2}{13} \)
9. \((5, 6), (9, 8)\) \( \frac{2}{4} = \frac{1}{2} \)
10. \((0, -4), (7, 3)\) \( \frac{7}{7} = 1 \)
11. \((-1, 7), (-3, 18)\) \( \frac{11}{-2} = -\frac{11}{2} \)
12. \((-6, -4), (1, 10)\) \( \frac{14}{7} = 2 \)

Adopted from McDougal Littell Inc. Geometry
Introduction to Geometry

FINDING THE EQUATION OF A LINE

Example 3
Find an equation of the line that passes through the point (3, 4) and has a y-intercept of 5.

\[ y = mx + b \]  \hspace{1cm} Write the slope-intercept form.

4 = 3m + 5  \hspace{1cm} Substitute 5 for b, 3 for x, and 4 for y.

-1 = 3m  \hspace{1cm} Subtract 5 from each side.

\[ \frac{-1}{3} = m \]  \hspace{1cm} Divide each side by 3.

The slope is \( m = \frac{-1}{3} \). The equation of the line is \( y = \frac{-1}{3} x + 5 \).

Exercises: Write the equation of the line that passes through the given point and has the given y-intercept.

13. \((2, 1); b = 5\)  \( y = \frac{-2}{3} x + 5 \)

16. \((7, 0); b = 13\)  \( y = \frac{-13}{7} x + 13 \)

14. \((-5, 3); b = -12\)  \( y = \frac{-3}{2} x + 3 \)

17. \((-3, -3); b = -2\)  \( y = \frac{1}{3} x - 2 \)

15. \((-3, 10); b = 8\)  \( y = \frac{5}{3} x + 8 \)

18. \((-1, 4); b = 8\)  \( y = \frac{1}{2} x - 8 \)

FINDING THE EQUATION OF A LINE

Example 4
Write an equation of the line that passes through the points \((4, 8)\) and \((3, 1)\). Find the slope of the line.

\[ m = \frac{8 - 1}{4 - 3} \]  \hspace{1cm} Substitute values.

\[ m = \frac{7}{1} = 7 \]  \hspace{1cm} Simplify.

1 = 7(3) + b  \hspace{1cm} Substitute values into \( y = mx + b \).

1 = 21 + b  \hspace{1cm} Multiply.

-20 = b  \hspace{1cm} Solve for \( b \).

Exercises: Write an equation of the line that passes through the given points.

19. \((6, -3), (1, 2)\)  \( y = -x + 3 \)

21. \((5, -1), (4, -5)\)  \( y = 4x - 21 \)

23. \((-3, -7), (0, 8)\)  \( y = 5x + 8 \)

20. \((-7, 9), (-5, 3)\)  \( y = -3x - 12 \)

22. \((-2, 4), (3, -6)\)  \( y = -2x \)

24. \((1, 2), (-1, -4)\)  \( y = 3x - 1 \)

Adapted from McDougal Littell Inc. Geometry
DISTANCE FORMULA

Example 5

Find the distance between the points (-4, 3) and (-7, 8)

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]
\[ = \sqrt{(-7 - (-4))^2 + (8 - 3)^2} \]
\[ = \sqrt{(-3)^2 + (5)^2} \]
\[ = \sqrt{34} \]

Exercises: Find the distance between the points

25. (3, 6), (0, -2) \[ \sqrt{73} \]
26. (5, -2), (-6, 5) \[ \sqrt{170} \]
27. (-3, 4), (1, 4) \[ 4 \]
28. (-6, -6), (-3, -2) \[ 5 \]
29. (8, -2), (-3, -6) \[ \sqrt{137} \]
30. (-8, 5), (-1, 1) \[ \sqrt{65} \]

COMBINING LIKE TERMS

Example 6

Simplify

\[ 8x^2 + 16xy - 3x^2 + 3xy - 3x \]
\[ 6x^2 - 3x^2 + 16xy + 3xy - 3x \]
\[ 5x^2 - 3x + 19xy \]

Group like terms

Simplify

Exercises: Simplify.

31. \[ 6x + 11y - 4x + y \]
\[ 2x + 12y \]
32. \[ -5m + 3q + 4m - q \]
\[ -m + 1q \]
33. \[ -3p - 4t - 5t - 2p \]
\[ -5p - 9t \]
34. \[ 9x - 22y + 18x - 3y \]
\[ 27x - 35y \]
35. \[ 3x^2y - 5xy^2 + 6x^2y \]
\[ 9x^2y - 5xy^2 \]
36. \[ 5x^2 + 2xy - 7x^2 + xy \]
\[ -2x^2 + 3xy \]

Adapted from McDougal Littell Inc. Geometry
SOLVING EQUATIONS WITH VARIABLES ON BOTH SIDES

Example 7
Solve.

\[ 6a - 12 = 5a + 9 \]

\[ a - 12 = 9 \]

Subtract 5a from both sides

\[ a = 21 \]

Add 12 to each side

Exercises: Solve the equation.

37. \[ 3x + 5 = 2x + 11 \]

\[ x = 6 \]

38. \[ 8m + 1 = 7m - 9 \]

\[ m = -10 \]

39. \[ 11q - 6 = 3q + 8q \]

No Solution

40. \[ -14 + 3a = 10 - a \]

\[ 4a = 24 \]

\[ a = 6 \]

41. \[ -3t + 10 = -t \]

\[ -t = -10 \]

\[ t = 10 \]

42. \[ -7x + 7 = 2x - 11 \]

\[ -9x = -18 \]

\[ x = 2 \]

SOLVING INEQUALITIES

Example 8
Solve.

a. \[ 5x - 4 \geq 4x + 6 \]

When you multiply or divide each side of an inequality by a negative number, you must reverse the inequality symbol to maintain a true statement.

b. \[ 10 - 7x < 24 \]

\[ -7x < 14 \]

\[ x > -2 \]

Exercises: Solve the inequality.

43. \[ -x + 2 > 7 \]

\[ -x > 5 \]

\[ x < -5 \]

44. \[ -5 + m < 21 \]

\[ m < 26 \]

45. \[ z + 6 > -2 \]

\[ z > -8 \]

46. \[ c - 18 < 10 \]

\[ c < 28 \]

47. \[ x - 5 < 4 \]

\[ x < 9 \]

48. \[ -3x + 4 \leq -5 \]

\[ -3x \leq -9 \]

\[ x \geq 3 \]
WRITING AND SIMPLIFYING RATIOS

Example 9

a. Train A takes 35 minutes to travel its route. Train B, traveling the same route but making more stops, takes 47 minutes. What is the ratio of the time of Train A to Train B?

b. Jennie’s height is 4 feet, 7 inches. Her younger sister’s height is 25 inches. Find the ratio of Jennie’s height to her sister’s.

Solutions

a. 35 minutes to 47 minutes = $\frac{35\text{ minutes}}{47\text{ minutes}} = \frac{35}{47}$

b. Convert 4 feet, 7 inches to inches: $4(12) + 7 = 55$ inches

$55\text{ inches} : 25\text{ inches} = \frac{55}{25} = \frac{11}{5}$

Exercises: Write the following ratios.

49. Basmati rice needs to cook for 20 minutes, while quinoa (another grain) cooks for 25 minutes. What is the ratio of cooking times for rice to quinoa? $\frac{20\text{ min}}{25\text{ min}} = \frac{4}{5}$

50. Jonathan caught 7 fish and Geogeanne caught 4. What is the ratio of fish caught of Jonathan to Geogeanne? $\frac{7}{4}$

51. Two sunflowers’ growth was measured daily. At the end of the experiment, Sunflower A had grown from 2 inches to 2 feet, 3 inches. Sunflower B had grown from 3 inches to 2 feet, 6 inches. Find the ratio of the growth in height of Sunflower A to Sunflower B. $\frac{25\text{ in}}{27\text{ in}} = \frac{25}{27}$

Use the diagram at the right.

52. What is the ratio of length to width of rectangle A? $\frac{4}{1}$

53. What is the ratio of the perimeter of rectangle A to the perimeter of rectangle B? $\frac{12}{8} = \frac{3}{2}$

54. What is the ratio of the area of rectangle A to the area of rectangle B? $\frac{16}{16} = 1$

DISTRIBUTIVE PROPERTY

Example 10

Solve.

a. $4(x + 3) = 36$

$4x + 12 = 36$

$4x = 24$

$x = 6$

b. $6(x + 4) + 12 = 5(x + 3) + 7$

$6x + 24 + 12 = 5x + 15 + 7$

$6x + 36 = 5x + 22$

$x = -14$
Exercises: Solve.

55. $2(x + 7) = 20 \quad x = -3$

56. $-10(y + 8) = 40 \quad y = -12$

57. $7(2 - x) = 5x \quad x = \frac{-14}{12} = \frac{-7}{6}$

58. $-4(x - 6) = 28 \quad x = -7$

SOLVING PROPORTIONS

Example 11

Solve.

a. $\frac{x}{8} = \frac{3}{4}$
   
   $4x = 8 \cdot 3$
   
   $4x = 24$
   
   $x = 6$

b. $\frac{6}{x+4} = \frac{1}{9}$
   
   $6 \cdot 9 = x + 4$
   
   $54 = x + 4$
   
   $50 = x$

Exercises: Solve.

59. $\frac{y}{50} = \frac{3}{100} \quad y = 1.5$

60. $\frac{6}{45} = \frac{2z + 10}{15} \quad z = -4$

61. $\frac{3}{p - 6} = \frac{1}{p} \quad p = -3$

62. $\frac{3}{8} = \frac{3}{2d} \quad d = 4$

63. $\frac{1}{18} = \frac{5}{-4(x - 1)} \quad x = -1.5$

64. $\frac{r}{3r + 1} = \frac{2}{3} \quad r = -\frac{2}{3}$

65. $\frac{3w + 6}{28} = \frac{3}{4} \quad w = 5$

66. $\frac{3}{m + 4} = \frac{9}{14} \quad m = \frac{3}{2}$

67. $\frac{w}{4} = \frac{9}{w} \quad w = \pm 6$

SIMPLIFYING RADICALS

Example 12

Simplify the expression $\sqrt{20}$

$\sqrt{20} = \sqrt{4} \cdot \sqrt{5}$

$= 2\sqrt{5}$

Exercises: Simplify the expression.

68. $\sqrt{121} = 11$

69. $\sqrt{400} = 20$

70. $\sqrt{243} = 9\sqrt{3}$

71. $\sqrt{52} = 2\sqrt{13}$

72. $\sqrt{27} = 3\sqrt{3}$

73. $\sqrt{288} = 12\sqrt{3}$

74. $\sqrt{45} = 3\sqrt{5}$

75. $\sqrt{80} = 4\sqrt{5}$

76. $\sqrt{320} = 8\sqrt{5}$

77. $\sqrt{72} = 6\sqrt{2}$

78. $\sqrt{50} = 5\sqrt{2}$

79. $\sqrt{225} = 15$

Adapted from McDougal Littell Inc. Geometry
SIMPLIFYING RADICAL EXPRESSIONS

Example 13

a. \[5\sqrt{3} - \sqrt{3} - \sqrt{2} = 4\sqrt{3} - \sqrt{2}\]

b. \[(2\sqrt{2})(5\sqrt{3}) = 2 \cdot 5 \cdot \sqrt{2} \cdot \sqrt{3} = 10\sqrt{6}\]

c. \[(5\sqrt{7})^2 = 5^2 \cdot \sqrt{7^2} = 25 \cdot 7 = 175\]

Exercises: Simplify the radical expression.

80. \[\sqrt{75} + \sqrt{3} = 6\sqrt{5}\]

81. \[-\sqrt{147} - \sqrt{243} = -16\sqrt{3}\]

82. \[(5\sqrt{3})(2\sqrt{4}) = 40\]

83. \[\sqrt{50} - \sqrt{18} = 2\sqrt{5}\]

84. \[(3\sqrt{14})(\sqrt{35}) = 21\sqrt{10}\]

85. \[(6\sqrt{5})^2 = 180\]

86. \[\sqrt{64} - \sqrt{28} = 8 - 4\sqrt{7}\]

87. \[(\sqrt{363})(\sqrt{300}) = 330\]

88. \[(4\sqrt{2})^2 = 32\]

89. \[\sqrt{44} + 2\sqrt{11} = 4\sqrt{11}\]

90. \[(\sqrt{32})(\sqrt{2}) = 8\]

91. \[(8\sqrt{3})^2 = 192\]

92. \[\sqrt{125} - \sqrt{80} = 5\]

93. \[(\sqrt{98})(\sqrt{128}) = 112\]

94. \[(10\sqrt{11})^2 = 1100\]

95. \[\sqrt{242} + \sqrt{200}\]

SIMPLIFYING QUOTIENTS WITH RADICALS

Example 14

Simplify the quotient \[\frac{6}{\sqrt{5}}\]

\[\frac{6}{\sqrt{5}} = \frac{6 \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} = \frac{6\sqrt{5}}{\sqrt{5}\sqrt{5}} = \frac{6\sqrt{5}}{5}\]

Exercises: Simplify the quotient.

96. \[\frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}\]

97. \[\frac{2\sqrt{3}}{\sqrt{5}} = \frac{2\sqrt{15}}{5}\]

98. \[\frac{\sqrt{32}}{\sqrt{5}} = \frac{4\sqrt{5}}{5}\]

99. \[\frac{\sqrt{12}}{\sqrt{7}} = \frac{\sqrt{84}}{7}\]

100. \[\frac{\sqrt{12}}{\sqrt{24}} = \frac{\sqrt{2}}{2}\]

101. \[\frac{\sqrt{27}}{\sqrt{45}} = \frac{\sqrt{5}}{5}\]

102. \[\frac{2\sqrt{3}}{\sqrt{6}} = \frac{\sqrt{2}}{1}\]

103. \[\frac{\sqrt{18}}{\sqrt{10}} = \frac{3\sqrt{2}}{5}\]

104. \[\frac{\sqrt{50}}{\sqrt{75}} = \frac{\sqrt{2}}{3}\]
SOLVING LITERAL EQUATIONS

Example 16

Given the formula for the surface area of a right cylinder, solve for $h$. $S = 2\pi r^2 + 2\pi rh$

$$S = 2\pi r(r + h)$$

$$\frac{S}{2\pi r} = r + h$$

or

$$\frac{S}{2\pi r} - r = h$$

Exercises: Solve the literal equation for the indicated variable. Assume variables are positive.

105. $V = \frac{4}{3} \pi r^3; r \quad r = \sqrt[3]{\frac{3V}{4\pi}}$

106. $V = s^3; s \quad s = \sqrt[3]{V}$

107. $V = \pi r^2 h; h \quad h = \frac{V}{\pi r^2}$

109. $P = 2l + 2w; l \quad l = \frac{P - 2w}{2}$

110. $S = 6x^2; s \quad s = \sqrt[3]{\frac{S}{6}}$

108. $A = \frac{1}{2}bh; h \quad h = \frac{2A}{b}$

112. $V = lwh; h \quad h = \frac{V}{lw}$

111. $A = \frac{1}{2}h(b_1 + b_2); b_1 \quad b_1 = \frac{2A}{h} - b_2$

ALGEBRAIC EXPRESSIONS

Example 17

a. Write an expression for seven less than a number $x$. $x - 7$

b. Write an equation for three times less than six times a number is five times the same number plus 5, then solve.

$6x - 3 = 5x + 5$

$x - 3 = 5$

$x = 8$

Exercises: Write the expression or equation. Solve the equations.

114. Half of a number plus three times the number

$$\frac{x}{3} + 3x = \frac{10x}{3}$$

115. The product of five and a number decreased by seven equals thirteen.

$$5(x - 7) = 13 \quad \Rightarrow \quad x = \frac{48}{5}$$

116. Sixteen less than twice a number is 10.

$$2x - 16 = 10 \quad \Rightarrow \quad x = 13$$

117. Twice a number increased by the product of the number and fourteen results in forty-eight.

$$2x + 14x = 48 \quad \Rightarrow \quad x = 3$$

118. Half of a number is three times the sum of the number and five.

$$\frac{1}{2}x = 3(x + 5) \quad \Rightarrow \quad x = -6$$
PERCENT PROBLEMS

Example 18

a. What number is 12% of 75?
   \[ x = 0.12(75) \]
   \[ x = 9 \]

b. 6 is what percent of 40?
   \[ 6 = 40p \]
   \[ 0.15 = p \]
   \[ p = 15\% \]

Exercises:

119. What number is 30% of 120?

   \[ 36 \]

120. 11 dogs is what percent of 50 dogs?

   \[ 22\% \]

121. What distance is 15% of 340 miles?

   \[ 51 \]

122. 200 is what percent of 50?

   \[ 400\% \]

123. 34 is what percent of 136?

   \[ 25\% \]

124. 8 weeks is what percent of a year?

   \[ \approx 15\% \]

SIMPLIFYING RATIONAL EXPRESSIONS

Example 19

Simplify.

a. \[ \frac{8x^2 + 12x}{4x^2 + 16x} = \frac{4x(2x+3)}{4x(x+4)} = \frac{2x+3}{x+4} \]

b. \[ \frac{y^2-9}{y^2+6y+9} = \frac{(y+3)(y-3)}{(y+3)(y+3)} = \frac{y-3}{y+3} \]

Exercises: Simplify.

125. \[ \frac{5x}{10x^2} = \frac{1}{2x} \]

126. \[ \frac{14d^2 - 2d}{6d^2 + 8d} = \frac{7d - d}{3d + 4} \]

127. \[ \frac{-5h + 1}{h + 1} = \frac{-5h + 1}{h + 1} \]

128. \[ \frac{16a^3}{8a} = 2a^2 \]

129. \[ \frac{2y - 12}{24 - 2y} = \frac{y - 6}{12 - y} \]

130. \[ \frac{t^2 - 1}{t^2 + 2t + 1} = \frac{t - 1}{t + 1} \]

131. \[ \frac{(5x^2 + x)}{(5x + 1)} = x \]

132. \[ \frac{36s^2 - 4s}{4s^2 - 12s} = \frac{9s - 1}{s - 3} \]

133. \[ \frac{m^2 - 4m + 4}{m^2 - 4} = \frac{m - 2}{m + 2} \]