

Students going into \Rightarrow *Advanced Placement Calculus (AB)*
Summer Packet Work {revised 2014}

ARE YOU READY FOR CALCULUS???

During the summer, we would like you to make sure you have reviewed the skills necessary to succeed in AP Calculus.

1. graphing calculator usage
2. algebra skills
3. trig skills
4. analysis of functions and their graphs

The AP test in Calculus requires that you have these graphing calculator skills:

1. Produce the graph of a function within an arbitrary viewing window.
2. Find the zeros of a function.
3. Compute the derivative of a function numerically.
4. Compute a definite integral numerically.

We will learn how to do 3 and 4 above during the school year, but you should do the following in the summer:

- Graph on the calculator: $y = x$, $y = x^2$, $y = x^3$, $y = \log x$, $y = \ln x$,
 $y = 2^x$, $y = e^x$, $y = |x|$, $y = x^{-1}$, $y = \frac{1}{x}$, $y = \frac{1}{x^2}$, $y = \sin x$, $y = \cos x$, $y = \tan x$,
 $y = \sec x$, $y = \csc x$, $y = \cot x$, and $y = \arctan x$. During the course of the year, you will be expected to produce these graphs without a calculator, as well.
- When graphing the trig functions, notice the difference using degree and radian mode. During the year, we will want to be in Radian mode when working with trig functions/expressions. However, in physics, you will probably be using Degree mode. Please be aware of this and careful to be in the correct mode in each class.
- Try creating windows that cover 1, 2 or 3 periods for the trig functions.
- Try graphing the other functions(non-trig) in a window which shows all important parts of the graph.
- As you look at the graphs, determine whether the functions are increasing, decreasing, concave up or concave down.
- State the domain and range of the functions.
- Find the roots or zeros of each function.
- Describe the asymptotic behavior and symmetry of each graph.
- Graph some of these graphs simultaneously and find their points of intersection.

NAME _____

Now for the algebra, trig and function material! This work should be done neatly on this paper, answers circled.

Simplify:

1. $\frac{x^3 - 9x}{x^2 - 7x + 12}$

2. $\frac{\frac{1}{x} - \frac{1}{5}}{\frac{1}{x^2} - \frac{1}{25}}$

3. $\frac{(x+1)^3(x-2) + 3(x+1)^2}{(x+1)^4}$

4. $\frac{\sqrt{1+x} - 1 - \frac{x}{2}}{x^2}$

5. $\log_2 5 + \log_2 (x^2 - 1) - \log_2 (x - 1)$

6. $3^{2\log_3 5}$

7. $\log 10^{\frac{1}{2}}$

Solve without a calculator:

8. $5^{(x+1)} = 25$

9. $\frac{1}{3} = 3^{2x+2}$

10. $\log_2 x = 3$

11. $\log_3 x^2 = 2\log_3 4 - 4\log_3 5$

12. $A = P + nrP$, for P

13. $8x^3 + 27 = 0$

14. $\frac{x+1}{x} - \frac{x}{x+1} = 0$

15. $|5x-2| = 8$

16. $|2x+1| = x + 3$

17. The equation $12x^3 - 23x^2 - 3x + 2 = 0$ has a solution of $x = 2$. Find all other solutions.

18. $x^2 + 2x - 3 < 0$

19. $\frac{2x-1}{3x-2} \leq 1$

Determine the equations of the following lines:

20. through $(-1, 3)$ and $(2, -4)$

21. through $(-1, 2)$ and perpendicular to the line $2x - 3y + 5 = 0$.

Factor completely:

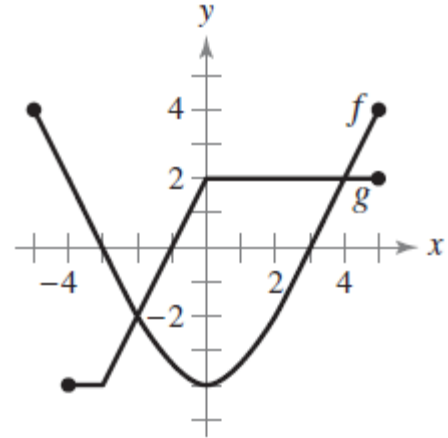
22. $x^6 - 16x^4$

23. $4x^3 - 8x^2 - 25x + 50$

24. Find the point of intersection of $3x - y - 7 = 0$ and $x + 5y + 3 = 0$.

25. The graphs of $f(x)$ and $g(x)$ are given:

- (a) State the values of $f(-5)$ _____ and $g(3)$ _____
- (b) for what values of x is $f(x)=g(x)$? _____
- (c) Estimate $f(-1)$ _____
- (d) On what interval is $f(x)$ decreasing? _____
- (e) State the domain of $f(x)$ _____
- (f) State the range of $f(x)$ _____



26. Find the remainder on the division of $x^5 - 4x^4 + x^3 - 7x + 1$ by $x + 2$.

27. Find the domain of the function $f(x) = \frac{3x+1}{\sqrt{x^2+x-2}}$.

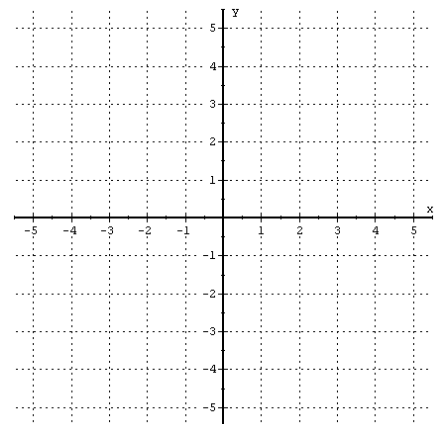
Find the domain and range of the functions:

28. $f(x) = 7$

29. $f(x) = \frac{5x-3}{2x+1}$

30 Let $f(x) = \frac{|x|}{x}$.

- (a) Graph $f(x)$
- (b) Rewrite as a piecewise function
- (c) Find the domain and range of $f(x)$.



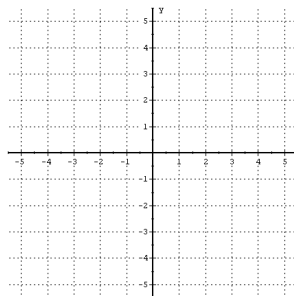
Simplify $\frac{f(x+h) - f(x)}{h}$ when $f(x) =$

31. $f(x) = 2x + 3$

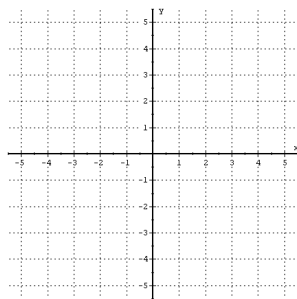
32. $f(x) = \frac{1}{x+1}$

33. $f(x) = x^2$

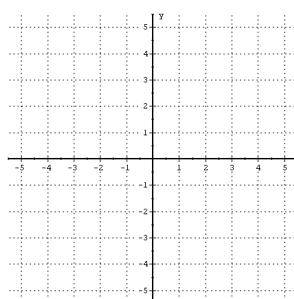
Graph the function $y = x^3$



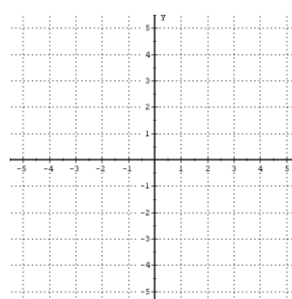
34. $f(x + 1) - 2$



35. $f(-x)$

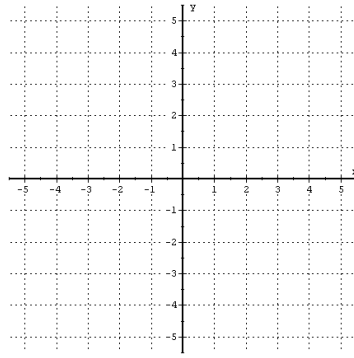


36. $|f(x)|$



Sketch the following graph:

37. $g(x) = |3x + 2|$



Write as a single equation in x and y:

38. $x = t + 1$
 $y = t^2 - t$

Find the inverse of the following functions:

39. $f(x) = 2x + 3$

40. $f(x) = \frac{x+2}{5x-1}$

41. $f(x) = x^2 + 2x - 1$, where $x > 0$

42. A water tank has the shape of a cone (like an ice cream cone, without the ice cream). The tank is 10 m high and has a radius of 3 m at the top. If the water is 5 m deep (in the middle), what is the surface area of the top of the water?

Without using a calculator, evaluate the following:

43. $\cos 210^\circ$

44. $\sin \frac{5\pi}{4}$

45. $\tan^{-1}(-1)$

46. $\text{Sin}^{-1}(-1)$

47. $\cos \frac{9\pi}{4}$

48. $\text{Sin}^{-1}\left(\frac{\sqrt{3}}{2}\right)$

49. $\tan \frac{7\pi}{6}$

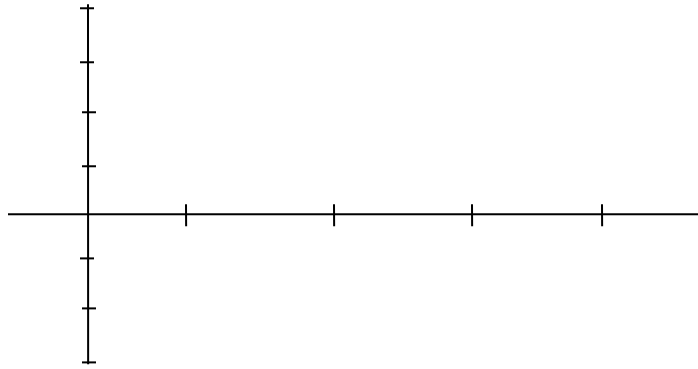
50. $\text{Cos}^{-1}(-1)$

51. Evaluate : $\cos 15^\circ$ without using a calculator

52. Evaluate : $\cos\left(\frac{\pi}{8}\right)$ without using a calculator

53. Solve $\sec^2(\mathbf{x}) - 2 \tan(\mathbf{x}) = 4$ for all values of 'x', in radians.

54. Graph: $y = -3\cos 2\left(x - \frac{\pi}{4}\right) + 1$
without using a calculator



Verify the following identities:

55. $\frac{\sec^2(x)}{\tan(x)} = \sec(x)\csc(x)$

56. $\tan^2(\theta) + 4 = \sec^2(\theta) + 3$

57. $\frac{\sin \alpha}{1 - \cos \alpha} = \frac{1 + \cos \alpha}{\sin \alpha}$

58. Be familiar with the **CALC** commands: value, root, minimum, maximum, intersect. You may need to zoom in on areas of your graph to find information. Answers should be accurate to 3 decimal places. Sketch the graph of $f(x) = 2x^4 - 11x^3 - x^2 + 30x$

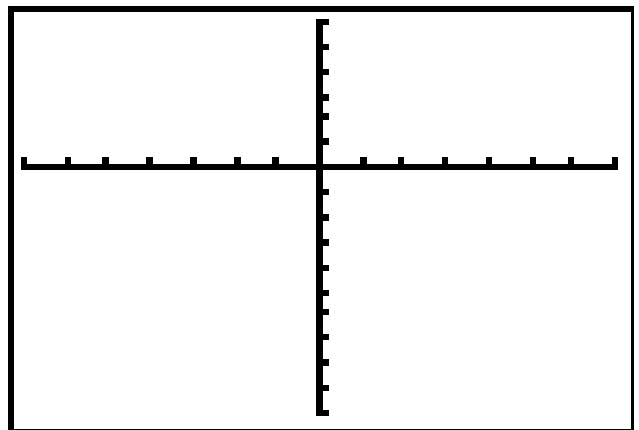
Window $xmin:-10, xmax: 10$ $scale: 1$
 $y-min: -100 ymax: 60$ $scale: 10$

(a) Find all roots _____

(b) Find all local Maxima _____

(c) Find all local Minima _____

(d) Find $f(-1)$ _____ $f(2)$ _____
 $f(0)$ _____ $f(.125)$ _____



Answers:

- (1) $\frac{x(x+3)}{x-4}$ (2) $\frac{5x}{5+x}$ (3) $\frac{x^2-x+1}{(x+1)^2}$ (4) $\frac{-1}{2\left[2\sqrt{1+x}+(2+x)\right]}$ (5) $\log_2(5x+5)$ (6) 25 (7) $\frac{1}{2}$ (8) 1 (9) $\frac{-3}{2}$ (10) 8 (11) $\pm \frac{4}{25}$ (12) $P = \frac{A}{1+NR}$
- (13) $\frac{-3}{2}$ (14) $\frac{-1}{2}$ (15) $\left\{2, \frac{6}{5}\right\}$ (16) $\left\{\frac{-4}{3}, 2\right\}$ (17) $\left\{\frac{-1}{3}, \frac{1}{4}, 2\right\}$ (18) (-3,1) (19) $\left(-\infty, \frac{2}{3}\right][1, \infty]$ (20) $7x+3y=2$ (21) $3x+2y=1$
- (22) $x^4(x+4)(x-2)$ (23) $(2x-5)(2x+5)(x-2)$ (24) (2, -1) (25) a. 4 & 2, b. -2 & 4, c. -3.5, d. (-5, 0) e. [-5, 5] f. [-4, 4] (26) -89 (27) $(-\infty, -2)(1, \infty)$ (28) $D: \mathbb{R}, R: y=7$ (29) $D: \mathbb{R} x \neq \frac{-1}{2}, R: \mathbb{R} y \neq \frac{5}{2}$
- (30) c. $D: \mathbb{R} x \neq 0, R: y=1$ or $y=-1$ (31) 2 (32) $\frac{-1}{(x+1)(x+h+1)}$ (33) $2x+h$ (38) $y = x^2 - 3x + 2$ (39) $f^{-1}(x) = \frac{x-3}{2}$
- (40) $f^{-1}(x) = \frac{x+2}{5x-1}$ (41) $f^{-1}(x) = -1 + \sqrt{x+2}$ (42) $\frac{9\pi}{4}$ (43) $\frac{-\sqrt{3}}{2}$ (44) $\frac{-\sqrt{2}}{2}$ (45) $\frac{-\pi}{4}$ (46) $\frac{-\pi}{2}$ (47) $\frac{\sqrt{2}}{2}$
- (48) $\frac{\pi}{3}$ (49) $\frac{\sqrt{3}}{3}$ (50) π (51) $\frac{\sqrt{2+\sqrt{3}}}{2}$ or $\frac{\sqrt{6+\sqrt{2}}}{4}$ (52) $\frac{\sqrt{2+\sqrt{2}}}{2}$ (53) $x = 1.249 + \pi n, \frac{-\pi}{4} + \pi n, -0.785 + \pi n$
- (58) a. -1.5, 0, 2, 5 b. (1.067, 20.101) c. (-.890, -18.483) & (3.948, -88.155) d. -18, 0, 0, 3.713